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LETTERS TO THE EDITORS

CONSERVATION LAWS FOR TWO-PHASE FLOW WITH A CHANGE OF PHASE

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ZUBER and Staub [1, 2] place much emphasis on using two equations of continuity for analysis of boiling, two-phase flow systems. They say that the great majority of analyses are incomplete in using only one equation of continuity, that is, the continuity equation for the mixture, instead of using two equations, one for each phase. Their statements cast doubt on a considerable amount of work in the analysis of two-phase flow. Consequently it is most important that the question be resolved of what constitutes a sufficient number of conservation equations.

Zuber and Staub derive a void propagation equation from three conservation equations, that is, continuity of liquid,*

$$\frac{\partial}{\partial t}[\rho_f(1-\alpha)] + \frac{\partial}{\partial z}[\rho_f(1-\alpha)v_f] = -\Gamma_g, \tag{1}$$

continuity of vapour,

$$\frac{\partial}{\partial t}[\rho_g\alpha] + \frac{\partial}{\partial z}[\rho_g\alpha v_g] = \Gamma_g, \tag{2}$$

and a particular form of the conservation of energy equation

$$(1-\alpha)\rho_f\frac{D_fE_f}{Dt} + \alpha\rho_g\frac{D_gE_g}{Dt} + \Gamma_g(E_g - E_f) = h\,\Delta T\left(\frac{\zeta_h}{A_c}\right) + \frac{\partial P}{\partial t} + \rho_m\frac{\partial\phi}{\partial t}. \tag{3}$$

The energy conservation equation used by most workers does not contain explicitly the term $\Gamma_g(E_g - E_f)$ and can be written

$$\frac{\partial}{\partial t}[\rho_fE_f(1-\alpha) + \rho_gE_g\alpha] + \frac{\partial}{\partial z}[\rho_fE_f(1-\alpha)\,v_f + \rho_gE_g\alpha v_g] = h\,\Delta T\left(\frac{\zeta_h}{A_c}\right) + \frac{\partial P}{\partial t} + \rho_m\frac{\partial\phi}{\partial t}. \tag{4}$$

For example, Kanai *et al.* [3] use this form of the energy equation but they ignore the kinetic and potential energy. We note that equation (4) could follow from equations (1–3).

* The Nomenclature of [1] is used throughout this note.

It is shown in this note that the void propagation equation of Zuber and Staub can be derived from the two equations the conventional conservation of energy equation (4) and the equation of continuity of the mixture,

$$\frac{\partial}{\partial t}[\rho_f(1-\alpha) + \rho_g\alpha] + \frac{\partial}{\partial z}[\rho_f(1-\alpha)v_f + \rho_g\alpha v_g] = 0. \tag{5}$$

Starting from equations (4) and (5) we firstly rewrite the energy equation as:

$$E_f\frac{\partial}{\partial t}[\rho_f(1-\alpha)] + E_g\frac{\partial}{\partial t}[\rho_g\alpha] + E_f\frac{\partial}{\partial z}[\rho_f(1-\alpha)v_f] + E_g\frac{\partial}{\partial z}[\rho_g\alpha v_g] = S, \tag{6}$$

where, by definition,

$$S \equiv h\,\Delta T\left(\frac{\zeta_h}{A_c}\right) + \frac{\partial P}{\partial t} + \rho_m\frac{\partial\phi}{\partial t} - \rho_f(1-\alpha)\frac{D_fE_f}{Dt} - \rho_g\alpha\frac{D_gE_g}{Dt}. \tag{7}$$

Multiplying the mixture equation by E_f and subtracting the result from equation (6) we obtain

$$\frac{\partial}{\partial t}[\rho_g\alpha] + \frac{\partial}{\partial z}[\rho_g\alpha v_g] = \frac{S}{E_g - E_f}. \tag{8}$$

Multiplying the mixture equation by E_g and subtracting the result from equation (6) we obtain

$$\frac{\partial}{\partial t}[\rho_f(1-\alpha)] + \frac{\partial}{\partial z}[\rho_f(1-\alpha)v_f] = \frac{-S}{E_g - E_f}. \tag{9}$$

Alternatively, equations (8) and (9) can be written, respectively,

$$\frac{\partial\alpha}{\partial t} + \frac{\partial}{\partial z}[\alpha v_g] = \frac{1}{\rho_g}\cdot\frac{S}{E_g - E_f} - \frac{\alpha}{\rho_g}\cdot\frac{D_g\rho_g}{Dt} \tag{10}$$

and

$$\frac{\partial\alpha}{\partial t} - \frac{\partial}{\partial z}[(1-\alpha)\,v_f] = \frac{1}{\rho_f}\cdot\frac{S}{E_g - E_f} + \frac{1-\alpha}{\rho_f}\cdot\frac{D_f\rho_f}{Dt} \tag{11}$$

Subtracting equation (11) from equation (10) we obtain

$$\frac{\partial j}{\partial z} = \frac{\rho_f - \rho_g}{\rho_f \rho_g} \cdot \frac{S}{E_g - E_f} - \frac{\alpha}{\rho_g} \cdot \frac{D_g \rho_g}{Dt} - \frac{1 - \alpha}{\rho_f} \cdot \frac{D_f \rho_f}{Dt}, \tag{12}$$

where, as in [1],

$$j = (1 - \alpha) v_f + \alpha v_g. \tag{13}$$

Returning to equation (10), the term $\partial/\partial z(\alpha v_g)$ can be written

$$\frac{\partial}{\partial z}[\alpha v_g] = \alpha \frac{\partial}{\partial z}[j + V_{gj}] + (j + V_{gj}) \frac{\partial \alpha}{\partial z}, \tag{14}$$

where, as in [1],

$$V_{gj} \equiv v_g - j. \tag{15}$$

With V_{gj} dependent only on α, as in [1],

$$\frac{\partial}{\partial z}[\alpha v_g] = \alpha \frac{\partial j}{\partial z} + \left(j + V_{gj} + \alpha \frac{\mathrm{d} V_{gj}}{\mathrm{d}\alpha}\right) \frac{\partial \alpha}{\partial z}. \tag{16}$$

Eliminating j and $\partial j/\partial z$ from equation (16) using equation (12) and substituting the result for $\partial/\partial z(\alpha v_g)$ into equation (10) we obtain

$$\frac{\partial \alpha}{\partial t} + \left\{ v_{fi} + V_{gj} + \alpha \frac{\mathrm{d} V_{gj}}{\mathrm{d}\alpha} + \int_0^z \left[\frac{\rho_f - \rho_g}{\rho_f \rho_g} \cdot \frac{S}{E_g - E_f} - \frac{1 - \alpha}{\rho_f} \cdot \frac{D_f \rho_f}{Dt} - \frac{\alpha}{\rho_g} \cdot \frac{D_g \rho_g}{Dt} \right] \mathrm{d}z \right\} \frac{\partial \alpha}{\partial z}$$
$$= \frac{\rho_m}{\rho_f \rho_g} \cdot \frac{S}{E_g - E_f} + \alpha(1 - \alpha) \left[\frac{1}{\rho_f} \cdot \frac{D_f \rho_f}{Dt} - \frac{1}{\rho_g} \cdot \frac{D_g \rho_g}{Dt} \right]. \tag{17}$$

Equation (17), the void propagation equation, is identical to equation (21) of [1]. The quantity $S/(E_g - E_f)$ can be seen, by inspection of equation (7), to be identical to Γ_g, the vapour source term given by equation (22) of [1].

We see that the results of [1] can be derived entirely from a single continuity equation for the mixture together with the conventional energy conservation equation. Two continuity equations, one for each phase, are not necessary. The additional equation is brought into the analysis of [1, 2] by the introduction of an additional variable, that is, Γ_g, the vapour source term. Identifying $S/(E_g - E_f)$ as the vapour source term we see that equations (8) and (9) are the continuity equations for the vapour and liquid phases, respectively. Thus the two continuity equations are implied in the mixture continuity and the conventional energy conservation equations.

Zuber and Staub [2] say that basic differences exist between their void propagation equation and the void propagation equation of Kanai *et al.* [3] who start, as in this note, from the mixture continuity and the conventional energy conservation equations. Kanai *et al.* consider two different assumptions for the relative velocity between vapour and liquid, firstly that slip ratio is dependent only on the void fraction and secondly that slip velocity is dependent only on the void fraction. For the first assumption, the differences observed by Zuber and Staub can be traced to a comparison of incompatible equations. Equation (59) of [2], an equation in the average void across the duct, should have been used instead of equation (30) of [2], an equation in the local void. For the second assumption, the differences can be traced to an erroneous equation (9) of [3].

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N. Spinks

Australian Atomic Energy Commission
Research Establishment
Lucas Heights, N.S.W.
Australia

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REJOINDER

In the introduction to his letter [1], N. Spinks states:

(1) Zuber and Staub [2] place much emphasis on using two equations of continuity for analysis of boiling two-phase flow.
(2) Consequently it is most important that the question be resolved of what constitutes a sufficient number of conservation equations.
(3) Zuber and Staub derive a void propagation equation from three conservation equations, that is, continuity of liquid, continuity of vapour and a particular form of the conservation energy equation.

Following the derivation of his equation (17), Spinks concludes: